Electromagnetic (I)
1st Midterm Exam.

College of Electronic Technology
Department of Communications

Date: 01/10/2022
Time: 90 min.

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Answer the following questions:

Q1) Consider the following vectors:

$\vec{A} = \hat{a}_x + 2\hat{a}_y - 3\hat{a}_z\,, \quad \vec{B} = -4\hat{a}_y + \hat{a}_z\,, \quad \vec{C} = 5\hat{a}_x - 2\hat{a}_z$

Find:

1. $\overrightarrow{a_A}$
2. $\overrightarrow{A}.\overrightarrow{B}$
3. $|\overrightarrow{A} - \overrightarrow{B}|$
4. θ_{AB}
5. $Prove\ that : \vec{A}.(\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}).\vec{C}$

Q2) Determine the area of a cylindrical surface described by:

$$\rho = 5m, 30^{\circ} \leq \emptyset \leq 60^{\circ}, 0 \leq z \leq 3m$$

As in the figure Q2

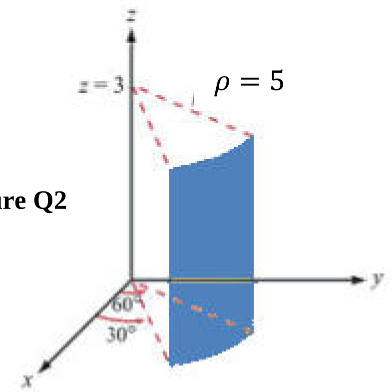


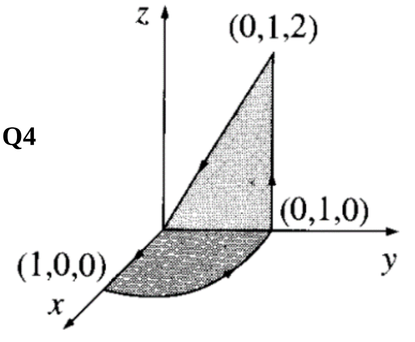
Figure Q2

Q3) Prove that the vector $\vec{A} = (x + y)\hat{a}_x + (y - x)\hat{a}_y + z\,\hat{a}_z$ in Cartesian coordinates is equal to $\vec{A} = \rho\hat{a}_\rho - \rho\hat{a}_\varphi + z\,\hat{a}_z$ in cylindrical coordinates.

Q4) The curl of the vector $\vec{G} = \rho(2 + sin^2\varphi)\hat{a}_\rho + \rho\sin(\varphi)\cos(\varphi)\,\hat{a}_\varphi + 3z\,\hat{a}_z$ is **equal to zero.**

Verify Stoke's theorem for the figure shown in Figure Q4.

Figure Q4



Vector Relations

Cartesian Coordinates (x,y,z)

$$\boldsymbol{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$d\vec{l} = \vec{a}_x dx + \vec{a}_y dy + \vec{a}_z dz$$
$$d\vec{l} = \vec{a}_\rho d\rho + \vec{a}_\phi \rho d\phi + \vec{a}_z dz$$
$$d\vec{l} = \vec{a}_r dr + \vec{a}_\theta r d\theta + \vec{a}_\phi r\sin\theta d\phi$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Cylindrical Coordinates (ρ,ϕ,z)

$$\boldsymbol{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho A_\rho\right) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

$$x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad z = z$$

Spherical Coordinates (r,θ,ϕ)

$$\boldsymbol{A} = A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(A_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \boldsymbol{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & (r\sin\theta)\mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & (r\sin\theta)A_\phi \end{vmatrix}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$x = \rho\cos\phi = r\sin\theta\cos\phi$$

$$y = \rho\sin\phi = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$\oint_L \mathrm{A}\cdot d\mathrm{l} = \int_S (\nabla\times\mathrm{A})\cdot d\mathrm{S}$$

$$\oint_S \mathrm{A}\cdot d\mathrm{S} = \int_v \nabla\cdot\mathrm{A}\, dv$$